Journal of Applied Mechanics and Technical Physics, No. 4, 1994

DISCHARGE OF A GAS-SATURATED LIQUID THROUGH A NOZZLE

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In this study we concern ourselves with the flow of a liquid in a nozzle when bubbles of dissolved gas form. The flow of a boiling liquid in channels was considered in [1, 2]. The flow of a liquid in a nozzle when individual gas bubbles appear in the stream was studied theoretically and experimentally in [3].

1. We consider the flow of a gas-saturated liquid at a pressure p_s in a varying-area channel. Because the pressure drops to $p \leq p_s$ the liquid may boil (a gas-vapor phase may form) in the stream because of dynamic processes. The following assumptions were made when constructing mathematical model in the quasi-one-dimensional approximation. A gas phase is formed only because of the literature of dissolved gas, accompanied by diffusion processes (the liquid is assumed to be cold and thus the partial vapor pressure of the liquid in the gaseous medium is ignored). Dissolved gas is released at impurity particles with an initial concentration n_0. Moreover, the phase velocities are equal and the forces of the friction of the stream against the channel wall is neglected. The temperature of the system is constant at T_0.

With the above assumptions the equations of conservation of mass for the liquid phase (parameters with a subscript) $(i = l)$ in the zone of two-phase flow have the form [1, 2]

$$\frac{\partial \rho_l F}{\partial t} + \frac{\partial \rho_l u F}{\partial z} = -nJF, \quad \frac{\partial \rho_g F}{\partial t} + \frac{\partial \rho_g u F}{\partial z} = nJF \tag{1.1}$$

$$\left(\rho_i = \rho_i^0 \alpha_i \ (i = l, g), \ \alpha_l + \alpha_g = 1, \ \alpha_g = \frac{4}{3}\pi a^3 n\right),$$

where $F = F(z)$ is the cross-sectional area of the channel; u is the velocity; ρ_i^0 and ρ_i are the true and average densities of the phases; α_i is the cubic content; a and n are the bubble radius and the number of bubbles per unit volume; and J is the rate of formation of the gas phase per inclusion. Combining Eqs. (1.1), we obtain the equation of conservation of mass for a two-phase mixture as a whole:

$$\frac{\partial \rho F}{\partial t} + \frac{\partial \rho u F}{\partial z} = 0 \ (\rho = \rho_l + \rho_g). \tag{1.2}$$

Moreover, we must write the equations of conservation of mass for the gas and liquid (solvent) dissolved in the liquid. The parameters pertaining to the liquid (solvent) and dissolved gas have a second subscript $j = l, g$. The equations of conservation of mass for the components are then written as

$$\frac{\partial \rho_{l(l)} F}{\partial t} + \frac{\partial \rho_{l(l)} u F}{\partial z} = 0, \quad \frac{\partial \rho_{l(g)} F}{\partial t} + \frac{\partial \rho_{l(g)} u F}{\partial z} = -nJF \tag{1.3}$$

$$(\rho_{l(j)} = \rho_{l(j)}^0 \alpha_l, \ \rho_l = \rho_{l(l)} + \rho_{l(g)}, \ \rho_l^0 = \rho_{l(l)}^0 + \rho_{l(g)}^0).$$

If we introduce the mass concentrations of the components

$$k_{(j)} = k_{l(j)} = \rho_{l(j)}/\rho_l = \rho_{l(j)}^0/\rho_l^0 \ (j = l, g),$$

then on the basis of Eqs. (1.1) and (1.3) we have

$$\rho_l \frac{dk_{(g)}}{dt} = -nJk_{(g)}, \ k_{(l)} = 1 - k_{(g)}. \tag{1.4}$$

Tyumen'. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, No. 4, pp. 91-99, July-August, 1994. Original article submitted May 25, 1993; revision submitted September 28, 1993.

0021-8944/94/3504-0569$12.50

The equation of conservation of the number of bubble inclusions when they do not subdivide or stick together is

$$\frac{\partial nF}{\partial t} + \frac{\partial nuF}{\partial z} = 0. \tag{1.5}$$

Then from (1.2) and (1.5) we easily obtain

$$\frac{d}{dt}\left(\frac{n}{\rho}\right) = 0,$$

whence

$$\frac{n}{\rho} = \frac{n_0}{\rho_{l0}^0} = \text{const}. \tag{1.6}$$

Here ρ_{l0}^0 and n_0 are the density of the initial liquid and the number of impurity particles per unit volume of the initial liquid.

The equation of conservation of momentum in the one-velocity approximation, neglecting the hydraulic resistance of the channel walls, has the form

$$\frac{\partial \rho uF}{\partial t} + \frac{\partial \rho u^2 F}{\partial z} = -F\frac{\partial p}{\partial z}. \tag{1.7}$$

The equation of state of the liquid is taken in the acoustic approximation and the gas is assumed to be calorically perfect:

$$p = p_0 + C_l^2(\rho_l^0 - \rho_{l0}^0),\ p = \rho_g^0 RT\ (T = T_0) \tag{1.8}$$

(R is the gas constant).

We assumed that the mass concentration of the dissolved gas for the saturated state satisfies Henry's law.

To complete the above equations we must give an expression for the gassing rate. For a gas stream diffusing toward the interface we assume that

$$J = 4\pi a^2 D \mathrm{Sh} \rho_l^0 \frac{k_{(g)} - k_{(g)a}}{2a}, \tag{1.9}$$

where D is the diffusion coefficient; Sh is the Sherwood number; $k_{(g)a}$ is the gas concentration as the interface that satisfies the saturation condition. In accordance with Henry's law we then write

$$k_{(g)a} = Gp\ (G = k_{(g)0}/p_s).$$

If the depletion of the dissolved gas in the carrier phase is ignored ($k_{(g)} = k_{(g)0}$), then Eq. (1.9) is rewritten as

$$J = 4\pi a^2 D \mathrm{Sh} \rho_{l0}^0 k_{(g)0} \frac{(p_s - p)/p_s}{2a}. \tag{1.10}$$

To assign the dimensionless mass-transfer coefficient Sh we assume that bubble growth due to the liberation of dissolved gas is described by the self-similar solution [2]. On the basis of the results of the self-similar solution of the problem of diffusion growth of bubbles in a liquid supersaturated with dissolved gas we write a relation for the Sherwood number:

$$\mathrm{Sh} = \frac{12}{\pi}\mathrm{Ja}\left[1 + \frac{1}{2}\left(\frac{\pi}{6\mathrm{Ja}}\right)^{2/3} + \frac{\pi}{6\mathrm{Ja}}\right],\ \mathrm{Ja} = \rho_{l0}^0 (k_{(g)} - k_{(g)a})/\rho_g^0.$$

2. For a steady-state flow Eq. (1.1) becomes

$$\frac{d\rho_l^0 a_l uF}{dz} = -nJF,\ \frac{d\rho_g^0 a_g uF}{dz} = nJF,$$

whereby it follows that

$$uF\left(\rho_l^0\alpha_l\frac{d\rho_g^0}{dz}+\rho_g^0\alpha_g\frac{d\rho_l^0}{dz}\right)+\rho_l\rho_g^0\frac{duF}{dz}=(\rho_l^0-\rho_g^0)nJF. \tag{2.1}$$

From the equation of mass (1.2) for the entire mixture as a whole we obtain

$$\rho uF = G = \text{const.} \tag{2.2}$$

We recast the equation of conservation of momentum (1.7) with allowance for (2.1) in the form

$$G\frac{du}{dz}+F\frac{dp}{dz}=0. \tag{2.3}$$

From (2.1) and (2.3), using the equation of state (1.8), we find

$$\frac{du}{dz}=\left(Q-\frac{u}{F}\frac{dF}{dz}\right)\left(1-\left(\frac{u}{C}\right)^2\right)^{-1},\ \frac{dp}{dz}=-\rho u\frac{du}{dz}$$
$$\left(Q=(1/\rho_g^0-1/\rho_l^0)nJ,\ C=\left(\frac{\rho\alpha_l}{\rho_l^0C_l^2}+\frac{\rho\alpha_g}{p}\right)^{-1}\right), \tag{2.4}$$

where C is the local "frozen-in" speed of sound. Neglecting the compressibility of the carrier liquid ($C_l=\infty$) and noting that in the process under consideration usually $\rho_g^0 << \rho_l^0$ and, therefore $\rho \approx \rho_l^0\alpha_l$ the local speed of sound is given by

$$C=\sqrt{\frac{p}{\rho_l^0\alpha_g\alpha_l}}.$$

With the above assumptions and further assuming that the volume gas content at the nozzle inlet is zero ($\alpha_{g0}=0$), on the basis of (1.6) and (2.2) we write

$$\frac{n}{1-\alpha_g}=n_0,\ (1-\alpha_g)uF=u_0F_0. \tag{2.5}$$

Here and below the subscript 0 pertains to the values of parameters at the nozzle inlet. Hence, taking into account the relation

$$\alpha_g=\frac{4}{3}\pi a^3 n,$$

we obtain an expression for α_g, n, and a in terms of the flow velocity:

$$\alpha_g=1-\frac{u_0F_0}{uF},\ n=n_0\frac{u_0F_0}{uF},\ a=\sqrt{\left(\frac{uF}{u_0F_0}-1\right)/\frac{4}{3}\pi n_0}.$$

Considering these expressions for the effect of gassing on the flow, we have

$$Q=\sqrt[3]{6\pi^2n_0^2\left[\left(\frac{u_0F_0}{uF}\right)^2-\left(\frac{u_0F_0}{uF}\right)^3\right]}D\ \mathrm{Sh\ Ja}.$$

Equation (1.4) for the variation of the concentration of the dissolved gas can be reduced to

$$u\frac{dk_{(g)}}{dz}=-\sqrt{6\pi^2n_0^2\left(\frac{uF}{u_0F_0}-1\right)}\frac{\rho_g^0}{\rho_l^0}D\ \mathrm{Sh\ Ja}.$$

On the basis of the above equations we make a qualitative analysis of the discharge of a gas-saturated liquid through a nozzle. Gassing begins in a cross section where the pressure in the flow reaches ρ_s. The parameters and coordinates of this cross section have a subscript s. Let us consider the asymptotic behavior of the growth of the bubble radius near the given cross.

From the second equation in (1.1) with allowance for (1.5) it follows that

$$u\frac{d}{dz}\left(\frac{4}{3}\pi a^3\rho_g^0\right) = J.$$

Near the cross section $z = z_s$ from this equation, using (1.10), we get the approximation

$$4\pi a^2 u_s \frac{da}{dz} = 2\pi a D \mathrm{Sh} k_{(g)0} \frac{\rho_l^0}{\rho_{gs}^0} \frac{p_s - p}{p_s}. \tag{2.6}$$

As $p \to p_s$ we have $\mathrm{Sh} \approx 2$. For the difference $p_s - p$ near the cross section where gassing begins we write

$$\frac{p_s - p}{p_s} = -\left(\frac{dp}{dz}\right)_s \frac{z_s - z}{p_s}. \tag{2.7}$$

Substituting (2.7) into (2.6), we obtain

$$a = \sqrt{\beta}\,(z - z_s),\ \beta = \frac{\rho_l^0}{\rho_{gs}^0} \frac{\rho_l^0 u_s}{p_s} \frac{1}{F_s} \left(\frac{dF}{dz}\right)_s Dk_{(g)0}.$$

This relation should be used in calculations from stationary equations upon passage through the cross section $z = z_s$. Moreover, the equation has a trivial solution $a = 0$, which corresponds to the absence of gassing (according to this solution, below the cross section z_s the gas-saturated liquid behaves as if it were in a "metastable" state).

The qualitative picture of the discharge of the system under consideration is similar to that of the discharge of boiling liquid through a nozzle [2]. Subsonic discharge may take place ($u < C$) under specific conditions at the nozzle inlet and exit.

Two fundamentally different critical discharge regimes are possible, depending on the properties of the gas-saturated liquid and the nozzle geometry. In the first case the transition from subsonic flow ($u < C$) to supersonic flow ($u > C$) occurs at the point with longitudinal coordinate $z_* < L$. Unlike the case of the flow of an ideal gas, when the throat (critical cross section) z_* is in the minimum cross section of the nozzle, for the flow of a gas-saturated liquid the throat is in the divergent section of the nozzle. This is because the effect of gassing on the flow is always positive ($Q > 0$). For the second regime, when the point z_* lies outside the channel ($z_* > L$), and integral line for which the flow velocity in the nozzle exit cross section ($z = L$) is equal to the speed of sound ($u = C$).

For gas-saturated liquids with gassing the critical flow rates corresponding to the given conditions for the pressure at the nozzle inlet and exit can be determined only by integration of the system of differential equations of motion of the medium. Moreover, this procedure enables us to determine the coordinate z_* of the throat and to take into account the effect of the "history" of the flow on the values of the parameters in the throat.

Let us consider the flow of a gas-saturated liquid where gassing (or dissolution) occurs so rapidly in the region of two-phase flow that the pressure in the stream follows the saturation pressure corresponding to the instantaneous concentration of dissolved gas ($p = p_s$). If the change in the instantaneous concentration of dissolved gas is ignored, then the region of two-phase flow in the stream is constant ($dp/dz = 0$). This scheme of flow corresponds to the formal solution of the system (2.4), when the number of nuclei is infinite ($n_0 = \infty$). Then from the equation of conservation of momentum we obtain

$$\rho u \frac{du}{dz} = 0,$$

and hence

$$u = u_s = \mathrm{const},\ p = p_s = \mathrm{const}. \tag{2.8}$$

From the equation of conservation of mass (2.5) with allowance for (2.8) we have

$$\alpha_g = 1 - F_s/F, \tag{2.9}$$

where F_s is the section where gassing begins.

In the region of two-phase flow, therefore, the volume content of the gas phase is determined unambiguously by the instantaneous value of the channel cross section. Since the volume content α_g is a nonnegative parameter ($\alpha_g \geq 0$), the cross section where the gassing begins should coincide with the nozzle throat ($F'(z) = 0$); henceforth F_s expresses the area of the minimum nozzle cross section. For the scheme of two-phase flow under consideration ($p = p_s = \text{const}$) the local speed of sound in the stream is zero ($C = 0$, $C^2 = dp/d\rho$), i.e., the flow corresponding to the solutions of (2.8), (2.9) formally is supersonic. If the pressure at the nozzle exit p_e is lower than the pressure when gassing begins ($p_e < p_s$), then the flow from the nozzle throat to the flow at the exit is described by the solution obtained. The contraction section of the flow is described by equations for an incompressible liquid, from which it follows that

$$p = p_0 + \frac{\rho_l^0 u_0^2}{2}\left[1 - \left(\frac{F_0}{F}\right)^2\right], \quad u = u_0 \frac{F_0}{F}. \tag{2.10}$$

For the critical discharge flow, determined by the condition for the pressure in the nozzle throat ($F = F_m$) to become equal to the pressure at which gassing begins ($p = p_s$) we obtain an expression for the critical flow rate:

$$q = F_0 u_0 = F_s u_s, \quad u_0 = \sqrt{\frac{2(p_0 - p_s)}{\rho_l^0\,[(F_0/F_s)^2 - 1]}}.$$

When the pressure p_e at the nozzle exit is higher than $p_s (p_e > p_s)$, for the diffusion section of the nozzle we can construct a discontinuous solution, changing the flow regime described by Eqs. (2.8) and (2.9) into the regime of one-phase flow of an incompressible liquid. The values of the parameters corresponding to the one-phase state behind the shock wave is labeled by the subscript f. Then the Bernoulli integral and the integrals of mass, written for an incompressible liquid, are valid for the flow behind the shock wave ($\alpha_g = 0$):

$$\frac{u^2}{2} + \frac{p}{\rho_l^0} = \frac{u_e^2}{2} + \frac{p_e}{\rho_l^0} = \frac{u_f^2}{2} + \frac{p_f}{\rho_l^0}, \quad \rho_l^0 uF = \rho_l u_e F_e = \rho_l u_f F_f. \tag{2.11}$$

Furthermore, at the shock wave we have the equations of conservation of mass and momentum:

$$\rho_l^0(1 - \alpha_{gf})u_s = \rho_l^0 u_f, \quad p_s + \rho_l^0(1 - \alpha_{gf})u_s^2 = p_f + \rho_l^0 u_f^2. \tag{2.12}$$

From this, with allowance for (2.11), we obtain an expression for the volume content of the gas phase α_{gf} ahead of the shock wave as a function of the pressure p_e at the nozzle exit:

$$\alpha_{gf} = \left\{1 - \left(\frac{F_s}{F_e}\right)^2 - \frac{p_e - p_s}{p_0 - p_s}\left[1 - \left(\frac{F_s}{F_0}\right)^2\right]\right\}^{1/2}.$$

The area F_f and coordinate z_f of the cross section as well as the pressure behind the shock wave can be determined from the equations

$$F_f = F_s/(1 - \alpha_{gf}), \quad F(z_f) = F_f, \quad p_f = p_s + \rho_l^0(1 - \alpha_{gf})\alpha_{gf} u_s^2, \tag{2.13}$$

which follows from (2.12).

For the pressure and velocity distributions behind the shock wave ($z > z_f$) from (2.11) and (2.13) we obtain

$$p = p_e + \frac{1}{2}\rho_l^0 u_s^2\left[\left(\frac{F_s}{F_e}\right)^2 - \left(\frac{F_s}{F}\right)^2\right], \quad u = u_0 \frac{F_0}{F}.$$

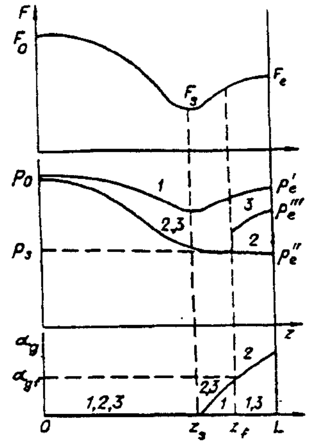

Fig. 1

Figure 1 schematically represents the above solutions, corresponding to a fixed inlet pressure. Line 1 corresponds to the case where the nozzle exit pressure is p_e' and the pressure p_s at which gassing begins is not reached in the channel; hence the flow is one-phase in the entire nozzle and the distribution of the parameters is found from (2.10). For line 2 the nozzle exit pressure is equal to $p_s(p_e'' = p_s)$, the flow is one-phase up to the cross section $F_s(0 < z < z_s)$ and two-phase after that cross section $(z_z < z < L)$. Lines 2 and 3 correspond to the critical discharge. In contrast to the case for line 2, $p_e''' > p_s$ for line 3. Lines 2 and 3 coincide up to the cross section with coordinate $z = z_f$. In the cross section $z = z_f$ the flow described by lines 2 jumps to a one-phase regime for which the pressure is p_e''' at the exit cross section.

When an equilibrium scheme is used to describe the flow of a gas-saturated liquid in the general case $(p = p_s)$ the pattern of the parameter distribution in the nozzle is completely analogous to that of the flow of gas streams. The speed of sound is determined from

$$C^2 = \chi \frac{\rho_l^0}{p_s} \frac{p^2}{\rho^2}, \rho = \rho_l^0 (1 - \alpha_g), \chi = \frac{\rho_g^0 (1 - k_{(g)0})}{\rho_l^0 k_{(g)0}},$$

3. As mentioned above, the construction of a solution of stationary equations describing the flow of gas-saturated liquids in a nozzle in a regime of critical discharge poses some difficulties because of the formation of sonic lines and shock waves. The calculations, therefore, were carried out on the basis of nonstationary equations by the fixing method. The solution was implemented numerically by the Laz–Bendorf method. Artificial diffusion was introduced into the calculations in order to smooth out the resulting jumps in the laws of conservation of mass and momentum [4]. The stationary equations make it possible to construct a solution for a subcritical regime of discharge and the nonstationary equations, for both critical and subcritical discharges. Accordingly, the fixing method was tested by comparing the solutions obtained from the stationary and nonstationary equations for the subcritical discharge regime.

The fixing method was tested for critical discharge regimes by varying the initial conditions. Two forms of conditions were considered: 1) there is no flow $(u(z) = 0)$, the pressure is the same everywhere $(p(z) = \text{const})$, and at a certain moment the nozzle is depressurized, i.e., the pressure at the exit drops $(p(L) = p_e)$; 2) an initial velocity is given at the nozzle inlet and the pressure distribution is found from the Bernoulli integral, with the exit pressure dropping at a certain moment. The initial density of the mixture was assumed to be equal to the density of the liquid and the size of the nuclei, which are discussed below, was determined against the background of the pressure obtained. The solutions for the two cases coincided completely.

A number of nuclei n_0 of initial size a_0 were introduced into the nozzle inlet. The size was chosen so that the time of self-similar growth of bubbles of this size would be substantially shorter than the time for which they remain in the nozzle $(t_D << t_*)$; the solution thus did not depend on the initial nucleus size. The calculations showed that $Ja \sim 10^2$-10^3. This can be a basis for estimating the time of self-similar growth $t_D \simeq a_0^2/2DJa^2$ for $Ja >> 1$.

On the basis of the equations given in Sec. 1 we calculated the discharge of carbonated water through a nozzle. As already mentioned, the method of calculation based on nonstationary equations can be used to find the distribution of the

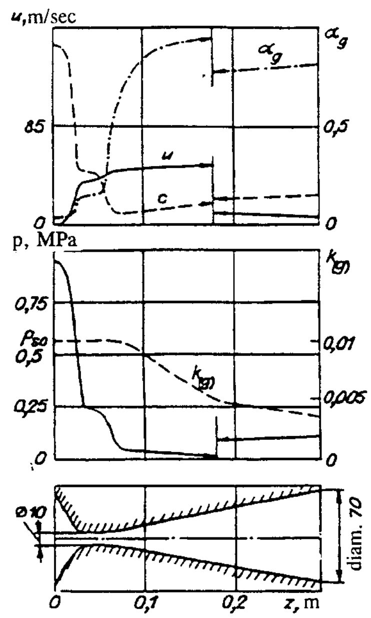

Fig. 2

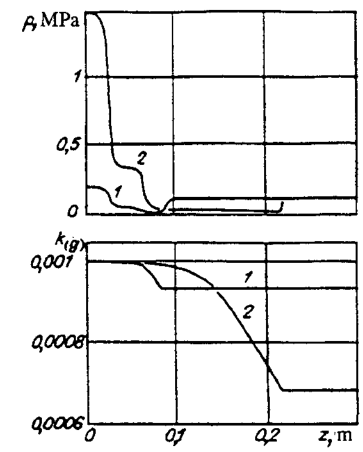

Fig. 3

parameters (pressure, flow velocity, speed of sound, gas saturation, and volume content of phases) along the entire length of the nozzle for both critical and subcritical discharge of a gas-filled liquid.

Figure 2 shows the stationary distribution of the average pressure p in the flow, the instantaneous pressure p_s of liquid saturation or the gas saturation $k_{(g)}$, the flow velocity u, the speed of sound C, and the volume content α_g of the gas phase in the critical discharge regime. The pressure is $p_0 = 1$ MPa at the nozzle inlet and $p_e = 0.1$ MPa at the exit. The following values were assumed for the parameters determining the physical properties of the liquid and the dissolved gas at the nozzle inlet: $\rho_{l0}{}^0 = 10^3$ kg/m^3, $C_l = 1500$ m/sec, $k_{(g)0} = 10^{-2}$, $p_s = 0.6$ MPa, $n_0 = 10^{13}$ m^{-3}, $a_0 = 10^{-5}$ m, $D = 10^{-9}$ m^2/sec, and $\rho_{gs}{}^0 = 1.815$ kg/m^3. As is seen from Fig. 2, near the beginning of the diffusion section of the nozzle the flow velocity reaches the speed of sound (u ~ 30 m/sec) in a mixture. Behind this cross section is a section where the flow is supersonic. Then in some cross section the flow abruptly becomes subsonic. Two zones can be distinguished in the supersonic flow portion of the nozzle: 1) a short section where the pressure drops abruptly; 2) a portion where the pressure drops slowly. The flow velocity increases further over the entire supersonic flow portion. At the point of transition to the subsonic regime the pressure increases abruptly, becoming roughly equal to the exit pressure p_e. In the supersonic flow section because of low pressure in the mixture the liquid is degassed rapidly and the concentration of dissolved gas drops sharply. Degassing is slower in the exit section of the nozzle, where the flow is already subsonic and the pressure in the mixture has risen.

Figure 3 illustrates the effect that the nozzle exit pressure has on the pressure and concentration distribution of the dissolved gas. Lines 1 and 2 correspond to $p_0 = 0.2$ and 1.5 MPa A rise in the exit pressure causes the supersonic–subsonic transition section to drift toward the nozzle exit, i.e., the zone of supersonic flow becomes longer. This is because the higher exit pressure results in a higher flow velocity. At some exit pressure (p_0 ~ 3.0 MPa) this section is beyond the nozzle exit.

Let us trace the effect of the inlet pressure on the concentration of the dissolved gas (or the instantaneous saturation pressure). At high concentrations of dissolved gas, when the saturation pressure of the liquid is substantially higher than the exit pressure ($p_s > p_e$), the inlet pressure does not have effect on the exit pressure of gas saturation. The reason for this is that degassing occurs in the zone of supersonic flow as well as in the zone of subsonic flow. The degassing rate, determined by the difference of the instantaneous saturation pressure and the pressure in the flow ($p_s - p$), is roughly the same because $p_s(z)$ is several times p(z) along the entire nozzle. For low concentrations (Fig. 3) ($p_s \le p_e$) gassing occurs only in the zone of supersonic flow. An increase in the exit pressure, therefore, causes greater degassing of the liquid since the degassing zone is enlarged.

Figure 4 shows the graphs of the distribution of the pressure (solid lines) in the flow and the gas saturation (dashed lines) when the nozzle exit pressure varied, with constant inlet pressure $p_0 = 0.6$ MPa. The exit pressure was $p_e = 0.1$ MPa in one case and $p_e = 0.2$ MPa in the other (lines 1 and 2). As follows from the graphs, a change in the exit pressure does not

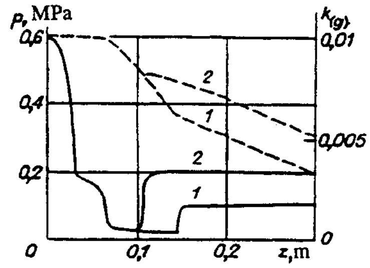

Fig. 4

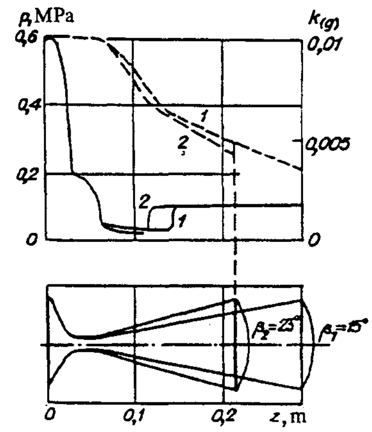

Fig. 5

affect the parameter distribution to the point of supersonic–subsonic transition. We also see that the decrease in exit pressure causes this point to drift toward the nozzle exit.

Figure 5 shows the graphs of the distribution of the pressure (solid lines) and the concentration of dissolved gas (dashed lines) for nozzles with different diffuser cone angles β = 15 and 23° (lines 1 and 2). The graphs indicate that an increase in the diffuser cone angle results in a somewhat lower pressure in the zone of supersonic flow. This is because the large diffuser cone angle leads to a large acceleration of the flow and thus a drop in pressure. Moreover, as the diffuser cone angle increases the supersonic-subsonic transition moves toward the throat. In other words, a reduction of the cone angle extends the zone of supersonic flow.

REFERENCES

1. O. E. Ivashnev and K. I. Soplenkov, "Nature of 'slow' rarefaction waves formed during discharge of a boiling liquid," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 3 (1991).
2. R. I. Nigmatulin, Dynamics of Multiphase Media. Parts 1, 2 [in Russian], Nauka, Moscow (1987).
3. J. T. Kuo and G. B. Wallis, "Flow of bubbles through nozzles," Int. J. Multiphase Flow, **14**, No. 5 (1988).
4. C. A. J. Fletcher, Computational Techniques for Fluid Dynamics, Vol. 1, Springer, Berlin (1988).